Housing Market with couples: When Distance Matters

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Introduction

How to allocate indivisible object in absence of monetary transfers?

- Allocating dorm rooms to students
- Allocating kidneys to patients
- Allocating public jobs to employees

Why not prices?

- Avoid the big role of the income!
- Social norms, legal and moral constraints

→ Need mechanisms/institutions different from competitive markets

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Outline

- Shapley Scarf Market: Housing Market

  → Key Solution Concepts/Mechanisms

- Shapley Scarf Market with Couples: When Distance Matters

  → Alternative Solution Concepts/Mechanisms

  → Further Research Questions/Open Questions

Shapley Scarf Market (Housing Market)

- I: Set of agents agents
- G: Set of goods with $|G| = |I|$
- $\succ_i$: Strict preference of i over goods
  $a \succ_i b$: a is better than b
- Agent $i \in I$ is initially endowed with a unique good
- Allocation: $\sigma : I \to G$
- Σ: Set of allocations
- σ^0: initial allocation
- $\sigma^0(i) = i$: the initial endowment of agent i

$\rightarrow$ Simplest possible exchange economy. Useful in analysis of several real world markets like market for houses, jobs, office spaces, kidneys for transplantation etc.

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Example

$$\begin{pmatrix} \succ_1 & \succ_2 & \succ_3 & \succ_4 \\ \hline 3 & 1 & 1 & 2 \\ 2 & 2 & 4 & 3 \\ 1 & 3 & 3 & 1 \\ 4 & 4 & 2 & 4 \end{pmatrix}$$

Central Planner

Preferences $\Longrightarrow$ Allocation Procedures $\Longrightarrow$ **Allocation**

Preferences $\Downarrow$

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	1	2
2	2	4	3
1	3	3	1
4	4	2	4

Allocation Procedures $\Downarrow$

?

Allocation $\Downarrow$

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	**1**	2
2	**2**	4	3
1	3	3	1
4	4	2	**4**

$\rightarrow$ Design "*good*" mechanisms to (re)allocate goods to the agents based on their preferences over goods.

Goals

Individual Rationality: An allocation σ is individual rational if every individual obtains a good that is at least as good as her initial good.

$$\sigma(i) \succcurlyeq \sigma^0(i) \text{ for all } i \in I$$

$$\begin{pmatrix} \succ_1 & \succ_2 & \succ_3 & \succ_4 \\ \hline \mathbf{3} & 2 & \mathbf{2} & 2 \\ 1 & \mathbf{1} & 4 & 3 \\ 2 & 3 & 3 & 1 \\ 4 & 4 & 1 & \mathbf{4} \end{pmatrix}$$

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$$\sigma(i) \succcurlyeq \sigma^0(i) \text{ for all } i \in I$$

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	2	**2**	2
1	**1**	4	3
2	3	3	1
4	4	1	**4**

Goals

Core: No group of agents can improve by breaking away and exchanging their initial endowments among themselves

An allocation σ is in the **Core** if there exists no coalition $S \subseteq I$ and no allocation $\bar{\sigma}$ such that:

1. $\bar{\sigma}(i) \in \{\sigma^0(i)\}_{i \in S}$ for all $i \in S$
2. $\bar{\sigma}(i) \succ \sigma(i)$ for all $i \in S$

$$\left(\begin{array}{cccc} \succ_1 & \succ_2 & \succ_3 & \succ_4 \\ \hline \mathbf{3} & 4 & \mathbf{2} & 2 \\ 1 & \mathbf{1} & 4 & 3 \\ 2 & 3 & 3 & 1 \\ 4 & 2 & 1 & \mathbf{4} \end{array}\right)$$

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$$\begin{pmatrix} \succ_1 & \succ_2 & \succ_3 & \succ_4 \\ \hline \mathbf{3} & 4 & \mathbf{2} & 2 \\ 1 & \mathbf{1} & 4 & 3 \\ 2 & 3 & 3 & 1 \\ 4 & 2 & 1 & \mathbf{4} \end{pmatrix}$$

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$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	4	**2**	2
1	**1**	4	3
2	3	3	1
4	2	1	**4**

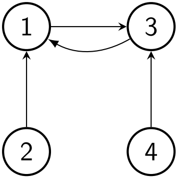
Gale's Top Trading Cycles Algorithm

Step 1: Each agent points to the owner of his favorite good. Since there are finite number of agents, there is at least one cycle. Each agent in a cycle is assigned the good of the agent he points to and removed from the market with his assignment. If there is at least one remaining agent, proceed with the next step.

......

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	**1**	3
2	4	4	2
1	3	3	1
4	2	2	4

Step 1:

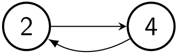


Gale's Top Trading Cycles Algorithm

Step t: Each remaining agent points to the owner of his favorite good among the remaining goods. Every agent in a cycle is assigned the good of the agent he points to and removed from the market with his assignment. If there is at least one remaining agent, proceed with the next step.

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	~~1~~	**1**	~~3~~
2	**4**	4	**2**
1	3	3	1
4	2	2	4

Step 2:



- The outcome of TCC is the unique core allocation (Roth and Postlewaite 1977)
- Core as a direct mechanism is strategy-proof (Roth 1982)
- Core is the only mechanism that is is individually rational, Pareto efficient, and strategy-proof (Ma 1994)

Shapley Scarf Market with Couples

- $\succ_i$: Strict preference of i over goods.
- $r(x, \succ_i)$: the rank of x in $\succ_i$.
- **Couples C** : $\{C_h, h = 1, ..., n/2\}$: The set of individuals is partitioned into couples.
  - $\sigma(C) = (\sigma(i), \sigma(j))$: the **ordered** set of goods that σ assigns to the members of $C = \{i, j\}$ where $i < j$.
- **Locations L** : a partition of G into non-empty subsets.
- Discrete metric d between goods (close or distant).
- Individuals compare allocations according to the **quality of their assigned good** and the **distance to partner**.

Preferences over Allocations

Standart S-S Market
$\mathcal{C} = \{\{1\},\{2\},\{3\},\{4\}\}$

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	1	2
2	2	4	3
1	3	3	1
4	4	2	4

S-S Market with Couples
$\mathcal{C} = \{\{1,2\},\{3,4\}\}$

R_1	R_2	R_3	p_4
$(3,1)$	$(3,1)$	$(1,2)$	$(1,2)$
$(2,1)$	$(3,2)$	$(1,3)$	$(3,2)$
$(3,2)$	$(1,2)$	$(3,4)$	$(3,4)$
...	...	...	...

Preferences over allocations

- The existence of couples generates interdependency in preferences.
- Rankings of goods must be extended to binary relations over allocations.
- No individual $i \in I$ pays attention to goods assigned to other couples.
- For all $i \in I$, **preferences over allocations:**
  - weak order R_i over $\mathbb{G} = \{(x, y) \in G \times G : x \neq y\}$.
  - $\sigma(C) P_i \overline{\sigma}(C) \Leftrightarrow i$ ranks allocation σ above allocation $\bar{\sigma}$.
    with a notational abuse, we write $\sigma R_i \bar{\sigma} \Leftrightarrow \sigma(C) R_i \overline{\sigma}(C)$.

Preferences over allocations

Interdependency through distance: $\forall \sigma, \bar{\sigma}$ and $\forall C = \{i, j\}$,

$$\left\{ \begin{array}{l} \sigma(C) = (\sigma(i), \sigma(j)) = (x, y) \\ \bar{\sigma}(C) = (\bar{\sigma}(i), \bar{\sigma}(j)) = (x, z) \\ d(x, y) = d(x, z) \end{array} \right\} \Rightarrow \sigma I_i \bar{\sigma}.$$

$C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, $L_1 =$ (1 2) and $L_2 =$ (3 4)

$$\begin{pmatrix} \succ_1 \\ \hline 2 \\ 3 \\ 1 \\ 4 \end{pmatrix}$$

$$\left\{ \begin{array}{l} \sigma = (3, 1, 2, 4) = (\sigma(1), \sigma(2), \sigma(3), \sigma(4)) \\ \bar{\sigma} = (3, 2, 1, 4) = (\bar{\sigma}(1), \bar{\sigma}(2), \bar{\sigma}(3), \bar{\sigma}(4)) \\ d(3, 1) = d(3, 2) = 1 \end{array} \right\} \Rightarrow \sigma I_1 \bar{\sigma}$$

Preferences over allocations

Separability w.r.t. good:

$$\left.\begin{array}{l} \sigma(C) = (\sigma(i), \sigma(j)) = (x, z) \\ \bar{\sigma}(C) = (\bar{\sigma}(i), \bar{\sigma}(j)) = (y, t) \\ d(x, z) = d(y, t) \text{ and } r(x, p_i) < r(y, p_i) \end{array}\right\} \Rightarrow \sigma P_i \bar{\sigma}.$$

$C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, L_1= (1 2) and L_2= (3 4)

$$\left(\begin{array}{c} \succ_1 \\ \hline 1 \\ 2 \\ 4 \\ 3 \end{array}\right)$$

$$\left.\begin{array}{l} \sigma = (1, 3, 2, 4) \\ \bar{\sigma} = (2, 4, 1, 3) \\ d(1,3) = d(2,4) = 1 \\ r(1, \succ_1) < r(2, \succ_1) \end{array}\right\} \Rightarrow \sigma P_1 \bar{\sigma}$$

Preferences over allocations

Separability w.r.t. distance:

$$\left\{\begin{array}{l} \sigma(C) = (\sigma(i), \sigma(j)) = (x, y) \\ \bar{\sigma}(C) = (\bar{\sigma}(i), \bar{\sigma}(j)) = (x, z) \\ d(x, y) < d(x, z) \end{array}\right\} \Rightarrow \sigma P_i \bar{\sigma}.$$

$C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, $L_1 =$ (1 2) and $L_2 =$ (3 4)

$$\begin{pmatrix} \succ_1 \\ \hline 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

$$\left\{\begin{array}{l} \sigma = (1, 2, 3, 4) \\ \bar{\sigma} = (1, 3, 2, 4) \\ d(1, 2) = 0 < d(1, 3) = 1 \end{array}\right\} \Rightarrow \sigma P_1 \bar{\sigma}$$

Preferences over allocations

For all individual i and for all allocation σ,
$(r(\sigma(i), \succ_i), d(\sigma(i), \sigma(j)))$ is an element of $\{1, ..., n\} \times \{0, 1\}$

$$\Downarrow$$

We can define R_i as weak orders $\trianglerighteq$ over $\{1, ..., n\} \times \{0, 1\}$ s.t.

- $[k < k'] \Rightarrow [(k, t) \triangleright (k', t)]$ for all t
- $(k, 0) \triangleright (k, 1)$ for all k

Example

- $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, $L_1 =$ (1 2) and $L_2 =$ (3 4)

$$\begin{pmatrix} \succ_1 \\ \hline 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

$\sigma = (1, 2, 3, 4)$, $\bar{\sigma} = (1, 3, 2, 4)$. Since $(1, 0) \rhd_1 (1, 1) \Rightarrow \sigma P_1 \bar{\sigma}$

$\sigma = (4, 3, 1, 2)$, $\bar{\sigma} = (2, 1, 4, 3)$. Since $(3, 0) \rhd_1 (4, 0) \Rightarrow \sigma P_1 \bar{\sigma}$

$\sigma = (4, 3, 1, 2)$, $\bar{\sigma} = (3, 1, 4, 2)$. $(3, 0) \rhd_1 (2, 1)$ or $(2, 1) \rhd_1 (3, 0)$

Example

- $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, $L_1 = (1 \quad 2)$ and $L_2 = (3 \quad 4)$

$$\begin{pmatrix} \succ_1 \\ \hline 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

$\sigma = (1, 2, 3, 4)$, $\bar{\sigma} = (1, 3, 2, 4)$. Since $(1, 0) \rhd_1 (1, 1) \Rightarrow \sigma P_1 \bar{\sigma}$

$\sigma = (4, 3, 1, 2)$, $\bar{\sigma} = (2, 1, 4, 3)$. Since $(3, 0) \rhd_1 (4, 0) \Rightarrow \sigma P_1 \bar{\sigma}$

$\sigma = (4, 3, 1, 2)$, $\bar{\sigma} = (3, 1, 4, 2)$. $(3, 0) \rhd_1 (2, 1)$ or $(2, 1) \rhd_1 (3, 0)$

Example

- $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$, $\quad L_1 =$ (1 2) and $L_2 =$ (3 4)

$$\begin{pmatrix} \succ_1 \\ \hline 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}$$

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$\sigma = (4, 3, 1, 2)$, $\bar{\sigma} = (3, 1, 4, 2)$. $(3, 0) \rhd_1 (2, 1)$ or $(2, 1) \rhd_1 (3, 0)$

Priority to Good Preferences

- $C_1 = \{1,2\}$ and $C_2 = \{3,4\}$
- L_1= (1 2) and L_2= (3 4)

$$\begin{pmatrix} \succ_1 \\ \hline 2 \\ 1 \\ 3 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} \rhd_1 \\ \hline (1,0) \\ (1,1) \\ (2,0) \\ (2,1) \\ (3,0) \\ (3,1) \\ (4,0) \\ (4,1) \end{pmatrix} \longrightarrow \begin{pmatrix} R_1 \\ \hline (2,1) \\ (2,3),(2,4) \\ (1,2) \\ (1,3),(1,4) \\ \ldots \end{pmatrix}$$

Priority to Distance Preferences

- $C_1 = \{1, 2\}$ and $C_2 = \{3, 4\}$
- L_1= 1 2 and L_2= 3 4

$$\begin{pmatrix} \succ_1 \\ \hline 2 \\ 1 \\ 3 \\ 4 \end{pmatrix} \longrightarrow \begin{pmatrix} \rhd_1 \\ \hline (1,0) \\ (2,0) \\ (3,0) \\ (4,0) \\ (1,1) \\ (2,1) \\ (3,1) \\ (4,1) \end{pmatrix} \longrightarrow \begin{pmatrix} R_1 \\ \hline (2,1) \\ (1,2) \\ (3,4) \\ (4,3) \\ (2,3),(2,4) \\ (1,3),(1,4) \\ (3,1),(3,2) \\ (4,1),(4,2) \end{pmatrix}$$

Core concepts

We impose potential blocking coalitions **not to break couples.**

- An allocation σ is in the **Core** ($\mathcal{C}(\mathcal{E})$) if there exists no $S \subseteq \mathbf{C}$ and an allocation $\bar{\sigma}$ s.t $\forall \{i, j\} = C \in S$,
  - $\bar{\sigma} P_i \sigma$ and $\bar{\sigma} P_j \sigma$
  - $\sigma(\mathbf{C} \setminus S) = \sigma^0(\mathbf{C} \setminus S)$.

The **Core** for the market ε is the subset $\mathcal{C}(\mathcal{E})$ of allocations which are not blocked.

Core concepts

We impose potential blocking coalitions **not to break couples.**

- An allocation σ is in the **Core** ($\mathcal{C}(\mathcal{E})$) if there exists no $S \subseteq \mathbf{C}$ and an allocation $\bar{\sigma}$ s.t $\forall \{i, j\} = C \in S$,
  - $\bar{\sigma} P_i \sigma$ and $\bar{\sigma} P_j \sigma$
  - $\sigma(\mathbf{C} \backslash S) = \sigma^0(\mathbf{C} \backslash S)$.

  The **Core** for the market ε is the subset $\mathcal{C}(\mathcal{E})$ of allocations which are not blocked.

Example

- $C_1 = \{1,2\}$, $C_2 = \{3,4\}$, L_1= (1 2) and L_2= (3 4)
- All agents have **priority to distance** preferences

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	1	2
2	2	4	3
1	3	3	1
4	4	2	4

$\longrightarrow$

R_1	R_2
$(3,4)$	$(2,1)$
$(2,1)$	$(1,2)$
$(1,2)$	$(4,3)$
$(4,3)$	$(3,4)$
...	...

$\sigma^0 = (1,2,3,4)$,
it's associated vectors for agents 1 and 2 are : $(3,0)$ and $(2,0)$
$\sigma^1 = (2,1,3,4)$
it's associated vectors for agents 1 and 2 are : $(2,0)$ and $(1,0)$
$\Longrightarrow \sigma^1 P_1 \sigma^0$ and $\sigma^1 P_2 \sigma^0 \quad \Longrightarrow \quad \sigma^0$ is not in the Core.

Example

- $C_1 = \{1, 2\}$, $C_2 = \{3, 4\}$, L_1= (1 2) and L_2= (3 4)
- All agents have **priority to distance** preferences

$$\left(\begin{array}{cccc} \succ_1 & \succ_2 & \succ_3 & \succ_4 \\ \hline 3 & {\color{red}1} & 1 & 2 \\ {\color{red}2} & {\color{blue}2} & 4 & 3 \\ {\color{blue}1} & 3 & {\color{blue}3} & 1 \\ 4 & 4 & 2 & {\color{blue}4} \end{array}\right) \longrightarrow \left(\begin{array}{cc} R_1 & R_2 \\ \hline (3,4) & {\color{red}(2,1)} \\ {\color{red}(2,1)} & {\color{blue}(1,2)} \\ {\color{blue}(1,2)} & (4,3) \\ (4,3) & (3,4) \\ \dots & \dots \end{array}\right)$$

$\sigma^0 = (1, 2, 3, 4)$,
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$\sigma^1 = (2, 1, 3, 4)$
it's associated vectors for agents 1 and 2 are : $(2, 0)$ and $(1, 0)$
$\Longrightarrow \sigma^1 P_1 \sigma^0$ and $\sigma^1 P_2 \sigma^0 \quad \Longrightarrow \quad \sigma^0$ is not in the Core.

Example

- $C_1 = \{1,2\}$, $C_2 = \{3,4\}$, $L_1 = (1 \quad 2)$ and $L_2 = (3 \quad 4)$
- All agents have **priority to distance** preferences

$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$
3	1	1	2
2	2	4	3
1	3	3	1
4	4	2	4

$\longrightarrow$

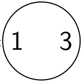
R_1	R_2
$(3,4)$	$(2,1)$
$(2,1)$	$(1,2)$
$(1,2)$	$(4,3)$
$(4,3)$	$(3,4)$
$\dots$	$\dots$

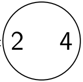
$\sigma^0 = (1,2,3,4)$,
it's associated vectors for agents 1 and 2 are : $(3,0)$ and $(2,0)$
$\sigma^1 = (2,1,3,4)$
it's associated vectors for agents 1 and 2 are : $(2,0)$ and $(1,0)$
$\Longrightarrow \sigma^1 P_1 \sigma^0$ and $\sigma^1 P_2 \sigma^0 \quad \Longrightarrow \quad \sigma^0$ is not in the Core.

Core concepts

- An allocation σ is in the **strict Core** ($\mathcal{C}^S(\mathcal{E})$) if there exists no $S \subseteq \mathbf{C}$ and an allocation $\bar{\sigma}$ s.t $\forall\{i, i'\} = C \in S$,
  - $\bar{\sigma} R_i \sigma$ and $\bar{\sigma} R_{i'} \sigma$ with at least one strict comparison
  - $\sigma(\mathbf{C} \backslash S) = \sigma^0(\mathbf{C} \backslash S)$.

We also consider another Core concept where blocking coalitions relate to a specific bargaining among partners.

- $C_1 = \{1, 2\}$, $C_2 = \{3, 4\}$, $L_1 =$ 

and $L_2 =$ 

R_1	R_2	R_3	R_4
$(4,1)$	$(2,1)$	$(3,4)$	$(1,2)$
$(3,4)$	$(4,3)$	$(1,2)$	$(3,4)$
$(2,1)$	$(2,3)$	...	...
$\dots$	$(1,4)$	$\dots$	$\dots$
	$(3,2)$		
	$(3,4)$		
	$\dots$		

$\sigma_1 = (3,4,1,2)$	$\sigma_2 = (2,1,3,4)$
$\max\{r(\sigma_1, R_1), r(\sigma_1, R_2)\} = 6$	$\max\{r(\sigma_2, R_1), r(\sigma_2, R_2)\} = 3$
$\min\{r(\sigma_1, R_1), r(\sigma_1, R_2)\} = 2$	$\min\{r(\sigma_2, R_1), r(\sigma_2, R_2)\} = 1$

$\Rightarrow$ σ_1 is min-max blocked by C_1 via σ_2

Core concepts

- An allocation σ is the **min-max Core** $\mathcal{C}^{Mm}(\mathcal{E})$ if there exists no $S \subseteq \mathbf{C}$ and $\bar{\sigma} \in \Sigma$ such that
  - $\forall \{i, j\} = C \in S$,

  $\max\{r(\bar{\sigma}, R_i), r(\bar{\sigma}, R_j)\} < \max\{r(\sigma, R_i), r(\sigma, R_j)\}$ and $\min\{r(\bar{\sigma}, R_i), r(\bar{\sigma}, R_j)\} < \min\{r(\sigma, R_i), r(\sigma, R_j)\}$

  - $\sigma(\mathbf{C} \backslash S) = \sigma^0(\mathbf{C} \backslash S)$.

Core concepts

- An allocation σ is the **strict min-max Core** $\mathcal{C}^{Mm}(\mathcal{E})$ if there exists no $S \subseteq \mathbf{C}$ and $\bar{\sigma} \in \Sigma$ such that
  - $\forall \{i, i'\} = C \in S$,

  $\max\{r(\bar{\sigma}, R_i), r(\bar{\sigma}, R_{i'})\} \leq \max\{r(\sigma, R_i), r(\sigma, R_{i'})\}$ and $\min\{r(\bar{\sigma}, R_i), r(\bar{\sigma}, R_{i'})\} \leq \min\{r(\sigma, R_i), r(\sigma, R_{i'})\}$ with at least one strict inequality

  - $\sigma(\mathbf{C} \backslash S) = \sigma^0(\mathbf{C} \backslash S)$.

Strict Minmax Core (SMm) $\subseteq$ Strict Core (Sco) $\subseteq$ Core (Co)
Strict Minmax Core (SMm) $\subseteq$ Minmax Core (Mm) $\subseteq$ Core (Co)

Main Results

All types of Core may be empty.

PRIORITY TO GOOD PREFERENCES

	Co	SCo	Mm	SMm
P.G	(TTC)	(TTC)	**???**	∅
P.G +all close			exist(TTC1)	∅

PRIORITY TO DISTANCE PREFERENCES

	Co	SCo	Mm	SMm
P.D	∅	∅	∅	∅
P.D+0-solvable	**???**	∅	**???**	∅
P.D+all close	(TTC1)	(TTC2)	(TTC1)	∅

0-solvable market: Given any initial distribution of goods among locations, trades can be organized so as to ensure that all partners are close.

THANK YOU!